Identification of Slowly Time-Varying Systems Based on The Qualitative Features of Transient Response A Frozen-Time Approach

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A method for structural and parameter identification of a slowly time-varying systems is proposed. The frozen-time method is used in this analysis. By means of this method we obtain consecutive LTI models, which are identified in consecutive discrete instants using the Qualitative System Identification (QSI) Algorithm. The proposed algorithm models the behavior of the ODE’s coefficients means of polynomial functions. The algorithm models the variations of those coefficients though polynomials. An optimal model is obtained using Genetic Algorithms. The algorithm starts with a polynomial of second degree and tries to fit these polynomials, to the variations of the coefficients. If the degree of the polynomials is not enough it increases and repeats the process until achieving a good fit. The system was tested with simulated experiments in matlab, and then tested with the identification of a controlled experiment in a power systems laboratory.

Keywords: Time-varying systems, LTI systems, Genetic Algorithms, Frozen-time approximation, Gradient optimization, System Identification.

1. Introduction

Practical systems are inherently time-varying, due to changes in operating conditions, drifting effects of components, on-line modeling processes, etc. One of the simplest and most tractable time-varying systems are slowly time-varying systems whose behavior resemble linear time invariant systems over a small period of time.

Slowly time-varying systems are of great importance in both practical applications and theoretical studies. Many practical systems are slowly time-varying. Environmental condition variations are usually much slower than systems dynamics. Therefore, a dynamic system with parameters dependent on the environment (temperature, pressure, altitude, etc.) can often be modelled as slowly varying systems. Component aging and deteriorations are another example of slow variations of systems dynamics in operation. Furthermore, non-linear systems operating on given trajectories can be modelled as linear time varying on those trajectories. When trajectories are sufficiently smooth, those systems become slowly varying systems.

One of the previous approaches for analyzing slowly varying systems is the frozen-time approach introduced in the 60’s [7,1], for stability analysis of systems with slowly time-varying parameters and used recently in [5] for identification and control. The main idea of the frozen-time approach can be summarized as follows: A time-varying plant is first modelled as a sequence of linear time-invariant systems, called frozen-time systems. The frozen-time system at time $t$ represents the dynamic behavior of the plant at that frozen time. At each frozen-time, the system’s identification process is carried out using the QSI software (see next section).

The resulting models of applying QSI, and the frozen-time approach, are organized consecutively forming a matrix that describes the behavior of the coefficients in time. The behavior of the coefficients of the ODE can be modelled independently by means of a polynomial function.
Section 2 presents how Qualitative System Identification (QSI) works. Section 3 we formulate the problem. Section 4 explain the system identification procedure proposed in this paper. Section 5 presents the pre-treatment and filtrate of the signal before processing it through QSI. Section 6 presents two application examples. Finally, section 7 presents the conclusions of this work.

2. Qualitative System Identification

QSI is a qualitative and quantitative system identification algorithm and software, developed by Flores and Pastor [3,10]. QSI takes as input a time-series representing the transient response of a LTI dynamic system and delivers a model of the identified system.

The identification algorithm of QSI is based on the fact that the response of a LTI system can be decomposed as a summation of exponential terms. If some of those exponentials terms are complex, in which case they are conjugate complex pairs, each pair forms a sinusoidal. Once that we can represent the behavior of this type of systems in terms of exponential and sinusoidal components, their response, we can make the following definitions:

\[ E_{n_1}(t) = \sum_{1 \leq i \leq n_1} C_i e^{-\tau_i t} \]  (1)

Equation 1 represents a sum of \( n_1 \) exponential terms,

\[ ES_{n_2}(t) = \sum_{1 \leq i \leq n_2} C_i e^{-\tau_i t} \sin(\omega_i t + \varphi) \]  (2)

and equation 2 represents a sum of \( n_2 \) damping sinusoidal functions.

The previous definitions allow to give a qualitative description of the behavior of the system from of the exponential and sinusoidal components, and it allows us to think about the following thing: a linear time-invariant system, of order \( n \), could be expressed as in the equation 3.

\[ y(t) = E_{n_1}(t) + ES_{n_2}(t) \]  (3)

where \( n_1 + 2n_2 = n \).

This result is evident from the definition of the equations 1 and 2.

If the second term of the equation 3 does not exist the response is non-oscillatory. In other case is a sinusoidal wave, where \( E_{n_1}(t) \) represents their attractor, and \( ES_{n_2}(t) \) is a damping sinusoidal component.

The algorithm is capable of separating the terms of Equation 3 to determine the structure or qualitative form of the system exhibiting the observed behavior. Separating the terms of the system’s response is performed by a filtering process. This process eliminates each component at a time, starting by the component with the highest frequency. Each time we eliminate one sinusoidal component, the remainder \( Y^*(t) \) contains the summation of all the previous components, except the eliminated one. After the elimination of \( j \) sinusoidal components, the remainder is:

\[ Y^*_j(t) = E_{n_1}(t) + ES_{n_2-j}(t) \]  (4)

The elimination of components continues until the rest of the signal is non-oscillatory. The remainder signal, after extracting the oscillatory components, is a summation of exponential terms, which are also identified and filtered one by one. Figure (1) shows a simplified version of the QSI algorithm [3]. QSI determines the order of the system by adding the order of all eliminated components. The filters TPAFilter and ExpFilter eliminate one by one each component and return the parameter of each component eliminated and the remainder signal.

<table>
<thead>
<tr>
<th>QSI(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>degree=0</td>
</tr>
<tr>
<td>P=0; Parameter Matrix</td>
</tr>
<tr>
<td>(X,k,P)=TPAFilter(X,degree,P)</td>
</tr>
<tr>
<td>(X,k,P)=ExpFilter(X,degree,P)</td>
</tr>
<tr>
<td>return Model(degree,X,P)</td>
</tr>
</tbody>
</table>

Fig. 1. QSI Algorithm

There are two main functions in the QSI algorithm: TPAFilter and ExpFilter. The function TPAFilter eliminates the sinusoidal components and returns the order corresponding to those components, the remainder signal and the parameters of the eliminated sinusoidal. The function ExpFilter eliminates the exponential components and returns the order of the model and the parameters. The remainder signal must be zero, since, all the components have been eliminated at this time.

At the same time that we eliminate each component, we isolate it to determine its parameters (quantitative or parametric identification), i.e., the coefficients of the ODE that models the observed system.

The QSI algorithm adds two units to the order of the system for each eliminated oscillatory component and one for each eliminated exponential component.
QSI determines the simplest LTI system capable of exhibiting the observed behavior. Equation 5 shows the form of the ODE obtained by QSI, that models a LTI system.

\[
\frac{d^n x}{dt^n} + C_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \cdots + C_0 x = 0
\]

(5)

where:

\[
C_{n-1} = \frac{a_{n-1}}{a_n}
\]

(6)

\[
C_{n-2} = \frac{a_{n-2}}{a_n}
\]

(7)

\[\vdots\]

\[
C_0 = \frac{a_0}{a_n}
\]

(9)

and \(a_n, a_{n-1}, \ldots, a_0\) are the coefficients of the general form of the ODE (see equation 10).

\[
a_n \frac{d^n x}{dt^n} + \cdots + a_1 \frac{dx}{dt} + a_0 x = 0
\]

(10)

3. Problem Formulation

The observation process is carried out by means of the frozen-time method, described previously. On each frozen-time instant the system’s transient response is captured and it is processed by QSI to obtain a LTI model. This process repeats for several consecutive moments, producing a set of differential equations that describes the behavior of the system for each instant of those frozen-times. With this set of ODEs we can form a matrix of coefficients that will allow us to observe their trends (see Table 1).

<table>
<thead>
<tr>
<th>(FT)</th>
<th>Coefficients of QSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(C_{n_1} = 1)</td>
</tr>
<tr>
<td>\vdots</td>
<td>(\vdots)</td>
</tr>
<tr>
<td>(k)</td>
<td>(C_{n_k} = 1)</td>
</tr>
</tbody>
</table>

The first column of Table 1 represents the frozen-time instants and the remainder columns the variation of the coefficients though time.

The equations characterizing time-varying systems are similar to those characterizing time-invariant systems, with the exception that the coefficients are functions of time. Thus time-varying systems are characterized by equation 11.

\[
a_n(t) \frac{d^n x}{dt^n} + a_{n-1}(t) \frac{d^{n-1} x}{dt^{n-1}} + \cdots + a_0(t) x = 0
\]

(11)

where

\[
a_n(t) = \sum_{j=0}^{D} p_j t^j
\]

(12)

and \(D\) is the highest degree of the polynomial that can model the variations of the coefficients. In other words; \(a_n(t)\) is a polynomial of degree \(D\).

We estimate the functions that approach these variations by means of genetic algorithms [9]. This method performs an optimization process in such a way that it adjust the behavior of the coefficients to polynomial functions. The algorithm begins with a polynomial of second degree and performs the optimization of this degree in such a way that the best approach is obtained.

4. The Identification Procedure

Build models using QSI and the frozen-time approach involves three basic elements; data, set of models, and functions that approach time-varying model. The data set are the time series captured from transient responses observed at each frozen instant. The set of models is obtained from processing this set of time series through QSI. The polynomials approach (see equation 12) are determined according with the algorithm proposed in Figure 2.

```
QSTimeVarying(D)
  [k,n]=size(D)
  do
    ren=ren+1
    mren=QSI(Xren)
  until ren=k
  ord=2
  do
    model=GeneticAlg(ord,M)
    r=valida(model)
    ord=ord+1
  until r > 0.9
  return model
```

Fig. 2. Time-varying system identification algorithm.

This algorithm works on data organized in a \(k \times N\) matrix where \(k\) is the number of frozen-instants and \(N\)
represents the size of the time series that captured the dynamics of each transient response for each frozen instant. Table 2 shows the organization of the data.

QSI identifies the models for each frozen instant, recording the obtained models in the coefficient matrix $M$.

$$M = \begin{bmatrix} C_{1,n} & C_{1,n-1} & \cdots & C_{1,0} \\ \vdots & \vdots & & \vdots \\ C_{k,n} & C_{k,n-1} & \cdots & C_{k,0} \end{bmatrix} \tag{13}$$

The columns of this matrix describes the behavior in the time of each coefficient of the characteristic equation. Since we have this set of models they are processed through a genetic algorithm to determine the function that best describes the behavior of the coefficients, i.e., we are identifying the functions $a_n(t)$, $a_{n-1}(t)$, ..., $a_0(t)$ of equation 11.

The first approach of the functions is made whit second order polynomials, if the approach doesn’t satisfy the criterion of a correlation coefficient $cc < 0.90$ then the process repeats increased in one the order of the polynomials until reaching it.

The validation process is performed in the following way: we evaluate the functions $a_n(t)$, $a_{n-1}(t)$, ..., $a_0(t)$ with $t = \{1, 2, \ldots, k\}$. We compute the correlation coefficient $cc_i$ between each resultant vector and their corresponding column in the matrix $M$. Finally $cc$ is obtained as the average of the $cc_i$'s.

The output of the algorithm is the matrix $M$, that best fits the observed data.

The application of this algorithm allow us obtain the coefficients of equation 11. The results section presents two applications cases of this methodology; the first problem is a simulated system in simulink, and the second problem is an identification experiment.

5. Data acquisition and pretreatment

When the data have been acquired in an identification experiment of a physical system, several factors can influence the acquisition of the data. In these cases it is to treat the signal before being processed by the identification algorithm. There are several potential problems in the data that need to be taken care of. These deficiencies are generally the product of a bad choice of the sampling interval, or simply of variations of the environment, and other types of external sources that affect the values that have been measured.

In off-line applications one may plot the data before processing them and inspect them looking for those deficiencies.

Those deficiencies can generate signals that are beyond our control and that affect the system. Inside the linear environment that is considered, it is assumed that those effects can be contained in an add-term $\epsilon(t)$ in the output of the system and which we usually call noise.

To pretreat the signal, a digital filter has been designed that allows us to eliminate the range of frequencies that alter their values. Digital filters are used mainly for two general purposes: separation of signals, for signals that has been mixed previously, and restoration of signals, for signals that have been distorted. The filter designed for this application is a low-pass filter of the type "FFT Convolution" [11]; this filter is used to separate a band of frequency from another. The strategy of the filter is simple, all frequencies below the cutoff frequency are passed with unitary gain, while all higher frequencies are blocked. Some of the ideal characteristics of this type of filters are:

- The magnitude of passband is perfectly flat, i.e., the width of the gain does not have oscillations that can alter the original value of the data.
- The attenuation in the stopband is infinite.
- The transition between the passband and stopband is infinitesimally small.

This filter works in frequency domain and their frequency response, $H(\omega)$, is given for next expresion

$$H(\omega) = \begin{cases} 1, & \text{if } |\omega| \leq \omega_c \\ 0, & \text{if } |\omega| > \omega_c \end{cases} \tag{14}$$

the Figure 3 shows the kernel of this filter.

By means of a convolucion process between the signal captured $x(t)$ and the filter kernel is possible to filter the signal in such a way that only the wanted frequencies are passed.
6. Results

In order to illustrate the benefits of this algorithm, we employ two cases. The first case is a mathematical representation of a mass-spring system simulated in Matlab. By this simulation we generated several transient responses and applied the identification algorithm.

The second case is a laboratory experiment representing a transmission line. In this experiment we simulate the aging of line by means of the increment of the resistance in each one of the observed frozen instants.

In both cases we observed and captured consecutive transient responses.

6.1. Correlation Coefficient

A measure to validate the obtained model is the correlation coefficient. For the application of this validation method it is necessary to carry out the simulation of the obtained model to compare its response with the observed signal. Equation 15 shows the expression to compute the correlation coefficient.

\[
cc^2 = \frac{\sum (\hat{y} - \bar{y})^2}{\sum (y - \bar{y})^2}
\]

(15)

\[
cc = \pm \sqrt{\frac{\sum (\hat{y} - \bar{y})^2}{\sum (y - \bar{y})^2}}
\]

(16)

where \(\hat{y}\) is estimated signal, \(y\) is observed signal and \(\bar{y}\) is the average of the observed signal.

The correlation coefficient \(cc\) compares the variance of the \(\hat{y}\) and \(y\). The correlation coefficient is confined inside the interval \([-1, 1]\]. When \(cc = 1\) represents a perfect positive correlation among the data. When \(cc = -1\) represents a perfect negative correlation, i.e., the data vary in opposed directions. When \(cc = 0\), there is not correlation among the data. The intermediate values describe partial correlations. For example if \(cc = 0.88\) means that the adjustment of the model to the data is reasonably good. In the practice, the values of \(cc\) are between 0 and 1.

6.2. Case I mass-spring system

Let us consider a mass-spring system with parameters that vary slowly with time, and whose model is described by the differential equation 17.

\[
m(t) \frac{d^2x}{dt^2} + Fr(t) \frac{dx}{dt} + K(t)x = 0
\]

(17)

Where \(m(t)\) is the mass, \(Fr(t)\) is the friction and \(K(t)\) is the spring constant. Assume the properties of the system change with time due to aging of components.

Let us apply the algorithm shown in Figure 2. The coefficient matrix obtained is presented in Table 3.

Note that in this paper we use uppercase letter \(K\) to denote the spring constant and the small letter \(k\) to denote a frozen-instant.

<table>
<thead>
<tr>
<th>(t_k)</th>
<th>(C_2(t_k))</th>
<th>(C_1(t_k))</th>
<th>(C_0(t_k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_5)</td>
<td>1</td>
<td>0.1642</td>
<td>1.9264</td>
</tr>
<tr>
<td>(t_7)</td>
<td>1</td>
<td>0.1846</td>
<td>2.1597</td>
</tr>
<tr>
<td>(t_9)</td>
<td>1</td>
<td>0.2086</td>
<td>2.5798</td>
</tr>
<tr>
<td>(t_1)</td>
<td>1</td>
<td>0.2121</td>
<td>2.9247</td>
</tr>
<tr>
<td>(t_4)</td>
<td>1</td>
<td>0.2469</td>
<td>3.3513</td>
</tr>
<tr>
<td>(t_6)</td>
<td>1</td>
<td>0.2826</td>
<td>3.744</td>
</tr>
<tr>
<td>(t_8)</td>
<td>1</td>
<td>0.3002</td>
<td>4.3944</td>
</tr>
<tr>
<td>(t_{10})</td>
<td>1</td>
<td>0.398</td>
<td>5.4008</td>
</tr>
<tr>
<td>(t_{11})</td>
<td>1</td>
<td>0.4755</td>
<td>6.4748</td>
</tr>
<tr>
<td>(t_{12})</td>
<td>1</td>
<td>0.5843</td>
<td>7.9399</td>
</tr>
<tr>
<td>(t_{13})</td>
<td>1</td>
<td>0.7543</td>
<td>9.9699</td>
</tr>
</tbody>
</table>

The set of models obtained by means of QSI have the form of monic polynomials; that is, the coefficient of the highest order term is unitary. Considering the model illustrated in equation 17, we assume that the coefficients of the characteristic equation, obtained by QSI, are given by:

\[
C_2(t_k) = \frac{m(t_k)}{m(t_k)} = 1
\]

(18)

\[
C_1(t_k) = \frac{Fr(t_k)}{m(t_k)}
\]

(19)

\[
C_0(t_k) = \frac{K(t_k)}{m(t_k)}
\]

(20)

Once the set of models is obtained for each frozen-instant \(k\), the algorithm estimate the functions that de-
scribe the behavior of each one of the coefficients. In this case, the given functions are described by a polynomial quotient, just as it is described in equations 19 and 20.

As it is described in section 3, the algorithm first proposes a quadratic polynomial to model the behavior of the coefficients. This is, the functions that describe these behaviors will be given by:

\[ m(t_k) = A_m t_k^2 + B_m t_k + C_m \tag{21} \]
\[ Fr(t_k) = A_{Fr} t_k^2 + B_{Fr} t_k + C_{Fr} \tag{22} \]
\[ K(t_k) = A_K t_k^2 + B_K t_k + C_K \tag{23} \]

We should determine the functions that correspond to \( m(t_k), Fr(t_k) \) and \( K(t_k) \), in such a way that equation 24 is minimized.

\[ dif(t_k) = \left( \frac{Fr(t_k)}{m(t_k)} - C_1(t_k) \right) + \left( \frac{K(t_k)}{m(t_k)} - C_0(t_k) \right) \tag{24} \]

\[ \sum dif^2(t_k) = 0 \tag{25} \]

The curve fitting was performed using two methods: genetic algorithms [9] and non linear least squares [2]. The obtained results are presented in the Table 4. In this case the quadratic functions given in equations 21, 22 and 23 were enough to model the behavior of the coefficients.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m(t) )</td>
<td>-0.003136</td>
<td>-0.003849</td>
<td>0.744887</td>
</tr>
<tr>
<td>( Fr(t) )</td>
<td>0.001056</td>
<td>0.006308</td>
<td>0.126297</td>
</tr>
<tr>
<td>( K(t) )</td>
<td>0.007828</td>
<td>0.169841</td>
<td>1.444667</td>
</tr>
</tbody>
</table>

Table 4

substituting the coefficients of the polynomials obtained by least squares in to equations 21, 22 and 23

\[ m(t) = -0.0031t^2 - 0.0038t + 0.74 \tag{26} \]
\[ Fr(t) = 0.00105t^2 + 0.0063t + 0.1242 \tag{27} \]
\[ K(t) = 0.0078t^2 + 0.1698t + 1.44 \tag{28} \]

and for the results obtained by genetic algorithms,

\[ m(t) = -0.0030t^2 - 0.0046t + 0.74 \tag{29} \]
\[ Fr(t) = 0.00109t^2 + 0.005t + 0.124 \tag{30} \]
\[ K(t) = 0.0079t^2 + 0.1661t + 1.45 \tag{31} \]

We substituted these results in equation 17 and solved the resulting ODEs, to compare among them and with the behavior exhibited by the simulated model.

Figure 4 shows both solutions of the estimated models and the response exhibited by simulated system.

The correlation coefficients between the response of the original system and the estimated models are \( cc = 0.99 \) with the model obtained by Genetic Algorithms and \( cc = 0.98 \) with the model obtained by Least Squares.

The results obtained in this identification experiment are satisfactory, since the validation tests reflect good results. The idea of carrying out the optimization using two methods was to have a comparison of the performance of the genetic algorithms versus a classic method. Figure 4 illustrates how similar the three signals are. In fact they are qualitatively identical. Also the correlation analysis reports values very near to unity.
6.3. Case II transmission line.

This experiment was performed in a power systems laboratory. The experiment consists in capturing the transient effect in a transmission line during the deconnection of the load, Figure 5 shows a), the single-phase diagram and b). The equivalent circuit. The equipment used was an experimental console LabVolt with an AC source of 20 volts; for the capturing the transient data we used the acquisition card of National Instruments NI PCI 5112, (100 MHz, 100 MS/s 8-Bit Digitizer). The model used in this test is the π model of the transmission line, this single-phase transmission line is shown in Figure 5. The values for the elements of this model were; \( V_s = 20v \), \( C_1 = 1.017 \mu F \), \( C_2 = 0.967 \mu F \), \( L = 29.65 mH \), and \( R \) varies as shown in Table 5.

![Fig. 5. Single-phase transmission line. b). Equivalent circuit.](image)

We used these laboratory devices to simulate a transmission line exhibiting the effects of aging. These effects were simulated by a variable resistor \( R \). We set the resistor to a given value (see Table 5), powered the transmission line and then disconnected the load. The transient effect was recorded. The experiment was repeated for the values of \( R \) shown in Table 5. During the experiments, the transient was recorded by measuring the voltage in \( C_2 \).

As we mentioned in section 5, during the data acquisition in an experiment, the data are not generally in good shape to be processed, and therefore it becomes necessary to pretreat them to eliminate noise and other components that can affect the identification process.

Figure 6 shows the acquired signal and the detail shows the result of the filtering process.

![Fig. 6. Acquired signal and detail of filtering process.](image)

Once the captured signals were filtered, we use the algorithm shown in Figure 2 to process this signal. Table 6 shows the matrix of coefficients produced by QSI.

The \( \pi \) model of a transmission line expressed as an ODE is shown in equation 32

\[
L(t) C(t) \frac{d^2 v_{c_2}}{dt^2} + R(t) C(t) \frac{dv_{c_2}}{dt} + v_{c_2} = v_s
\]

therefore

\[
C_2(t_k) = \frac{L(t_k) C(t_k)}{L(t_k) C(t_k)} = 1
\]

\[
C_1(t_k) = \frac{R(t_k) C(t_k)}{L(t_k) C(t_k)} = \frac{R(t_k)}{L(t_k)}
\]

<table>
<thead>
<tr>
<th>( t )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{k1} )</td>
<td>0.386</td>
</tr>
<tr>
<td>( t_{k2} )</td>
<td>0.396</td>
</tr>
<tr>
<td>( t_{k3} )</td>
<td>0.406</td>
</tr>
<tr>
<td>( t_{k4} )</td>
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</tr>
<tr>
<td>( t_{k5} )</td>
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</tr>
<tr>
<td>( t_{k6} )</td>
<td>0.476</td>
</tr>
<tr>
<td>( t_{k7} )</td>
<td>0.516</td>
</tr>
<tr>
<td>( t_{k8} )</td>
<td>0.596</td>
</tr>
<tr>
<td>( t_{k9} )</td>
<td>0.776</td>
</tr>
<tr>
<td>( t_{k10} )</td>
<td>1.216</td>
</tr>
</tbody>
</table>
Table 6
Coefficients of a transmission line experiment

<table>
<thead>
<tr>
<th>$t_k$</th>
<th>$C_2 (t_k)$</th>
<th>$C_1 (t_k)$</th>
<th>$C_0 (t_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>1</td>
<td>11.337800349</td>
<td>32147873.02</td>
</tr>
<tr>
<td>$t_2$</td>
<td>1</td>
<td>11.631528978</td>
<td>31834739.03</td>
</tr>
<tr>
<td>$t_3$</td>
<td>1</td>
<td>11.925245445</td>
<td>32076947.89</td>
</tr>
<tr>
<td>$t_4$</td>
<td>1</td>
<td>12.51270542</td>
<td>31982739.73</td>
</tr>
<tr>
<td>$t_5$</td>
<td>1</td>
<td>12.806430888</td>
<td>32087805.38</td>
</tr>
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<td>1</td>
<td>13.9813328</td>
<td>32087805.78</td>
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<tr>
<td>$t_7$</td>
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<td>31955757.71</td>
</tr>
<tr>
<td>$t_8$</td>
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</tr>
<tr>
<td>$t_9$</td>
<td>1</td>
<td>22.79309717</td>
<td>32087809.44</td>
</tr>
<tr>
<td>$t_{10}$</td>
<td>1</td>
<td>35.71701824</td>
<td>32018492.74</td>
</tr>
</tbody>
</table>

$C_0 (t_k) = \frac{1}{L (t_k) C (t_k)}$ (35)

The genetic algorithm must determine the functions for $R (t)$, $L (t)$ and $C (t)$ in such a way that the equation 36 is minimized.

$$df (t_k) = \left( \frac{R (t_k)}{L (t_k)} - C_1 (t_k) \right) + \left( \frac{1}{L (t_k) C (t_k)} - C_0 (t_k) \right)$$ (36)

$$\sum df^2 (t_k) = 0$$ (37)

Following the procedure described in section 3, the genetic algorithm fits tests fitness with second degree polynomials. As the approach provided by the second degree polynomials is not enough to give a good fit to the data, the algorithm increases the order and repeats. This process is carried out repeatedly until achieving a good fit. Finally the best fit is achieved by fourth degree polynomials; Figure 7 presents the approaches provided by the tested polynomials for the coefficient $C_1 (k)$.

The polynomials that best describe the behavior of the data of Table 6 are the following:

$$R (t) = 4.74E^{-4} t^4 - 61E^{-4} t^3$$ (38)
$$+ 2.6E^{-2} t^2 - 2.8E^{-2} t + 26.8E^{-2}$$

$$L (t) = -1.0E^{-7} t^4 - 3.0E^{-7} t^3$$ (39)
$$+ 2.0E^{-7} t^2 - 5.7E^{-5} t + 2.3E^{-2}$$

$$C (t) = -1.5E^{-11} t^4 + 4.0E^{-10} t^3$$ (40)
$$- 3.0E^{-9} t^2 + 1.0E^{-8} t + 1.33E^{-6}$$

As we can observe in the functions given by equations 39, and 40, the coefficients of the terms from the first to the fourth order they are small compared to the independent term. That is to say, these terms do not contribute significantly in the evaluation of their respective functions. In practical terms we can assume that those functions are constant. Figure 8 presents the graphs corresponding to $R (t)$, $L (t)$ and $C (t)$. The continuous lines represent the estimated values and the dashed lines represent the expected values.

Fig. 7. Three different fittings for the coefficient $C_1 (k)$.

Fig. 8. Comparing between expected values and estimated values for $R (t)$, $L (t)$, $C (t)$.

We can observe that the lines corresponding to $L (t)$ and $C (t)$ are practically horizontal lines, compared with the variation of $R (t)$.
Figure 9 shows the signal observed in a frozen-instant, \( k \), and their corresponding simulated signal after the identification process. We compute the coefficients using equations 38, 39, and 40 evaluated at instant \( k \).

The correlation coefficient between the model estimated and the expected model is \( cc = 0.9988 \). I.e., the model that we estimate reproduces the dynamics of the system appropriately.

7. Conclusions

In this paper, a new identification algorithm for slowly time-varying systems has been proposed. The algorithm is based on the QSI and frozen-time approaches. We pretreated the signal using a low-pass filter to eliminate the inherent noise to the laboratory measurements. The two examples of application of this method were satisfactory, since the models can reproduce the dynamics of the systems with great accuracy. The validation tests of the estimated models are acceptable, with correlation coefficients very near to unity. The algorithm was validated at simulation level using matlab and simulink, and also with real laboratory measurements.

References


